Journal of Mathematical Sciences: Advances and Applications
Volume 4, Number 1, 2010, Pages 61-72

THE ANALYSIS OF AN EFFICIENT ALGORITHM BASED ON GAME THEORY APPLIED TO THE DETECTION OF MEDICAL ERRORS

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Abstract

In order to improve upon self-detection of medical errors, a medical network is represented as a directed tree graph. For each practitioner in the network and every potential error, a utility function is assigned in order to estimate the management risk involved in delaying the check of that error. Using this utility function, efficient algorithms are designed for checking all the practitioners of a network in a manner which minimizes the total utility cost of errors. Methods and principles from game theory and linear programming are utilized in the algorithm design.

1. Introduction

Human errors, both deliberate and accidental, are an inexorable part of all facets of human behavior. These errors are especially significant in the field of medical practice. Despite the fact that this problem existed since the beginning of medical practice, in recent years it has attracted

2010 Mathematics Subject Classification: 91A80.

Keywords and phrases: game theory, algorithm, efficient, medical, errors, physicians.

Received November 14, 2008

a great deal of attention and research. In the report, To Err Is Human: Building a Safer Health System, the Institute of Medicine (IOM) estimates that preventable medical errors are the eighth leading cause of death among Americans, causing 44,000 to 98,000 deaths each year, at an annual cost of approximately $17 billion [11].

The intricacies of this problem and the vast amount of possible solutions have been addressed from many different perspectives. Various solutions include the following ideas [1], [2], [3], [11] and [17]: (i) educating medical personnel regarding the significant adverse impact that results from medical errors; (ii) training medical personnel in methods of avoiding medical errors in the first place, and in detecting and correcting these errors in the case that they do occur; (iii) incorporating defense mechanisms into the medical system to prevent errors from harming the patient; (iv) designing computer databases that organize and analyze medical data, thereby preventing human errors [15]; (v) creating risk management strategies [7]; and (vi) designing self-detection systems ensuring review by experienced practitioners.

The goal of this article is to expand on the solution of self-detection systems. The creation of a hierarchical network of a medical organizational evaluation structure is suggested. This structure consists of administrators, physicians, nurses, clerks, etc., who are divided into several levels. The upper level consists of the medical administrators and head physicians, who supervise and evaluate the medical personnel under their guidance. The physicians at the next level, in turn, evaluate the practitioners below them, and so on until the bottom of the structure. Except for the highest and lowest levels, each person is checked by others above him, and simultaneously checks those under his supervision. Additionally, this network has a managing committee that ensures that everyone is performing their respective duties of supervision. This network is a self-evaluating system that unifies all the participants into a complete and all-inclusive evaluation process. The formal definition of this network and its representation as a directed tree graph are discussed in Section 2.

Given a network, the task of ordering the checks of its practitioners presents itself. The order in which these checks are performed is significant because: (i) time constraints limit the number of nurses that could be checked; (ii) negative effects of errors worsen over time; (iii) uncorrected errors have a tendency to recur; and (iv) errors propagate downward through the network. To order these checks in a manner which minimizes the error, a utility function is defined and assigned to each practitioner, relative to each error. This function gives a rough estimate of the risk brought about by delaying the check of a given error made by a given practitioner. The specifics of this function are discussed in Section 3.

Once the utility function is defined, an algorithm is developed to minimize the error by determining the most efficient order for the checks. Two cases are treated: (i) the probabilities of the errors are known; and (ii) the probabilities of the errors are unknown. The algorithm for case (i) is direct, while the algorithm for case (ii) is based upon principles and methods of game theory and linear programming. These algorithms are fully elucidated in Section 4. Directions of future research are presented in Section 5.

2. The Network

A network is modeled with a directed tree graph consisting of L different levels (See Diagram 1). Let $N_k =$ the number of medical personnel in the k-th level of the network. For each $0 \leq k \leq L$, and $0 \leq j \leq N_k$, let $D_{k,j}$ denote the j-th node on the k-th level, which represents the j-th medical personnel on the k-th level of the network. An edge directed from $D_{k,j}$ to $D_{k+i,j}$ indicates that $D_{k,j}$ is in charge of supervising $D_{k+i,j}$. For each level k, let $M_k =$ the number of errors that the practitioners of level k are checked for. These errors are denoted by $E_{1,}\ E_2, \ldots, E_{M_k}$.

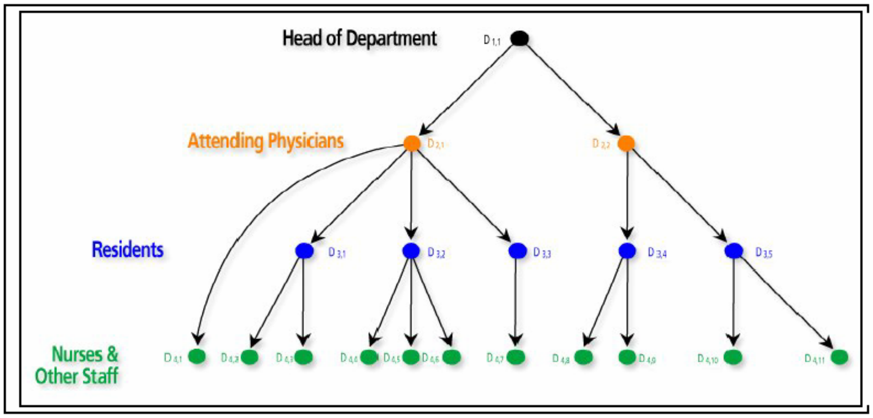

Diagram 1. A sample medical network represented by a directed tree graph.

3. The Utility Function

To order these checks in a manner that minimizes the total error, we define a utility function and assign it to each practitioner relative to each error. This function provides a rough estimate of the risk brought about by delaying the check of a given error made by a given practitioner. For each medical practitioner $D_{k,j}$ and error E_i, the utility function $U(i, j, k)$ is defined as follows:

$$U(i, j, k) = \frac{W(i, j, k) \cdot C(i, j, k) \cdot F(i, j, k) \cdot R(i, j, k) \cdot Q(i, j, k) \cdot S(i, j, k)}{T(i, j, k)}.$$

The functions of which the utility function is composed are defined as follows:

1. $W(i, j, k)$: Weight Function: This function measures the negative *physical* effect incurred by delaying the check of error E_i. It is measured from the perspective of the loss of quality of life, the loss of functionality, and the pain involved. Clearly, an error that causes severe damage is more pressing than an error which causes minor discomfort. This function can be obtained from the cybernetics study of public health [19], [20].

2. $C(i, j, k)$: Cost Function: This function measures the *monetary* expenses incurred by delaying the check of error E_i. Clearly, an error which costs a hospital millions of dollars should be checked before one with little financial loss. This function can be modelled from the theory of risk aversion [4], [9], [12].

3. $F(i, j, k)$: Frequency Function: This function measures the number of patients the error effects. Clearly, a greater frequency creates a greater urgency to correct the error.

4. $R(i, j, k)$: Recurrence Function: This function measures the tendency of a given error to repeat. A high tendency to recur creates a strong urgency to correct.

5. $Q(i, j, k)$: Quality Function: This function measures the quality of each practitioner and his relative likelihood to make error E_i. The purpose of this function is to compensate for the fact that statistically determined error probabilities are independent of the quality and experience of particular practitioners. This function is carefully determined based upon the statistical analysis and discretion of the administrators of a particular network [6], [12].

6. $S(i, j, k)$: Shift Function: This function measures the effect of the particular shift, in which a procedure is performed upon the error probability. For instance, it is known that procedures performed during shift changes, in the middle of the night, or on holidays are more likely to contain errors.

7. $T(i, j, k)$: Time Function: This function measures the amount of time that is necessary to check for error E_i. The significance of this function is based upon the realization that assuming that all other factors are equal, errors that take a shorter time to check should be given preference over errors that take a longer time to check.

We are currently working with radiologists from the University of Pittsburgh Medical Center in order to define these functions precisely. We are collecting data regarding errors and consulting medical experts and literature to properly evaluate the significance of various factors in the effects of medical errors. Although, we have mentioned qualitative factors which impact the effect of errors, assigning appropriate quantities to these factors within the weight function is a significant challenge, and requires much consideration and analysis of data. For the purposes of this paper, however, we will assume that the definition and values of these functions and the utility function are known and will describe our system and algorithms based upon these values.

4. Algorithms

The goal is to construct an algorithm that provides an efficient way of ordering the procedure of checking doctors in a manner, which minimizes the total cost. Since, errors propagate downward in a network, and an error at a higher level can impact all the levels below it, checking higher levels is given priority to checking lower levels. Within each level class, the utility function $U(i, j, k)$ is used to determine the order of the checks. In order to illustrate, the algorithms of ordering the practitioners on the same level, the following problem will serve as an example.

Given. A resident is charged with checking 4 nurses for the following 3 errors: dispensing a drug at the wrong time, serving inappropriate food, (i.e., bad for patient's diet) and administering the wrong drug.

Data. The chart below reflects the values of the utility functions of the nurses regarding these 3 errors.

Question. How should the resident order the checking of the nurses in order to reduce the total cost of the errors?

Utility function for:	Nurse 1	Nurse 2	Nurse 3	Nurse 4
Wrong time of drug	100	200	300	1000
Wrong food	0	100	700	600
Wrong drug	300	500	400	1100

<u>Solution</u>

Each level k is represented by a matrix M, where the rows represent the errors E_i, and the columns represent the doctors $D_{k,j}$.

The matrix entries are defined by the utility functions defined above, i.e., $m_{i,j} = U(i, j, k)$. The goal is to define the utility function for all the doctors on level k and order the doctors $D_{k,j}$ in such a way that minimizes this utility function. The solution is divided into two methods.

Method I

Assume that a statistical analysis of empirical data determines that the probability of each type of error E_i is given by q_i. Assume that these probabilities will remain the same in the future. The algorithm is as follows:

1. For each $D_{k,j}$, compute the expected error $r(k, j) = \sum_{i=1}^{M_k} m_{i,j} \cdot q_i$.

2. Order the $D_{k,j}$'s by ordering the values of $r(k, j)$ in descending order. Thereby those practitioners with the largest utility functions are checked first, and the total error is minimized.

3. If two or more nurses have the same value for $r(k, j)$, then we order them lexicographically.

Method II

If the probabilities q_i of each E_i are not given, then an algorithm is constructed based upon principles from game theory. The probabilities of the errors are treated as objective variables independent of any particular doctor. Being that the statistical data is subjective, statistical data is not used to determine these probabilities. However, data is used to determine the effect of the subjective competence of the practitioner $D_{k,j}$ regarding error E_i, upon the utility function, $U(i, j, k)$.

Consider the model as a two-person, zero-sum game, where the players are *society* and *the forces of evil*. The objective is to find a combination of probabilities q_i, that minimizes the expected error [13], [14], and [16].

Let M be the matrix with elements $m_{i,j}$.

Definition. Column j *dominates* column l, if $m_{i,j} \geq m_{i,l}$ for all i.

Definition. An entry that is minimal for its row and maximal for its column is called a *saddle point*. This corresponds to a nurse, whose worst error is the least costly of all other nurses.

To find saddle points:

(i) For each column j, compute its maximum value, and call it s_i.

(ii) Compute the minimum of all the s_i.

Thus, the saddle point $SP_j = Min_j(Max_i m_{i,j})$.

There could be several columns with saddle points, but all the saddle points will have the same numeric value. Each such column is removed from the matrix.

In game theory, two strategies are used: pure strategy (strictly determined game) and mixed strategy (non- strictly determined game). In a strictly determined game, the player chooses a pure strategy of only one particular option.

The difficulty in applying methods from game theory to the problem of ordering checks for errors is that game theory determines which move to make, and is not concerned with the other moves. Therefore, the standard algorithm involves the removal of rows and columns. However, checking for errors demands an ordering of *all* the columns. Therefore, an algorithm is designed to order the columns of M in a way that guarantees that no row is removed at any stage.

Algorithm

Begin with the original matrix M with M_k rows and N_k columns.

Pure strategy

1. Check to see that none of the columns of M dominate each other.

If one of the columns dominates the other column, then remove that column from M.

2. Check to find a saddle point $SP(i_s, j_s)$ in M.

For each saddle point $SP(i_s, j_s)$, remove column j_s from M.

Mixed strategy

3. Use linear programming to solve for the mixed strategy solution for M.

The *Fundamental Theorem of Game Theory* guarantees the existence of a solution to every two player zero-sum game. However, such a solution may have zeroes as some of the probabilities for the errors [14]. To solve this problem, the solutions are restricted by imposing a lower bound (ε) on all the probabilities of the errors. Let $\vec{w}$ be a vector with components all 1's. Let $\nu = \sum_{i=1}^{M_k} y_i$. The following linear programming problem must be considered:

Minimize ν subject to the constraints $M\vec{y} \geq w$ and $\vec{y} \geq \varepsilon$.

By letting $z_i = y_i - \varepsilon$, this can be transformed to an equivalent problem:

Minimize ν subject to the constraints $M\vec{z} \geq w_z$ and $z \geq 0$, where $w_z = (I - \varepsilon M)\vec{w}$.

In order for, this problem to be have meaning it is required that $\varepsilon = \min_{1 \leq j \leq N_k} (\sum_{i-1}^{M_k} m_{i,j})$ over all j.

Additionally, ε be small enough such that the linear programming generates the best approximation to the solution of the probabilities of the errors.

4. Once these probabilities are obtained, apply Method I to order *all* the columns of the original matrix, including those which were removed because of saddle points or dominance.

Application of algorithm

Consider the example of the resident charged with checking 4 nurses for errors E_1, E_2, and E_3. Begin with the following matrix:

E_1 = Wrong time of drug
E_2 = Wrong food
E_3 = Wrong drug

E_1	100	200	300	1000
E_2	0	100	700	600
E_3	300	500	400	1100
	$D_{k,1}$	$D_{k,2}$	$D_{k,3}$	$D_{k,4}$

1. Nurse $D_{k,2}$ dominates over Nurse $D_{k,1}$. Therefore, column 2 is removed from the matrix.

2. Column 1 has a saddle point and is, therefore, removed as well.

3. Use linear programming on the rest of the matrix to find the probabilities q_i. This gives:

$$q_1 = \frac{53\varepsilon}{8(\varepsilon + 1)}; \quad q_2 = 0.875; \quad q_3 = \frac{1 - 52\varepsilon}{8(\varepsilon + 1)}.$$

4. Then apply Method I to the original matrix with these q_i' s. This yields:

$$r_1 = \frac{3 - 103\varepsilon}{8(1 + \varepsilon)}; \quad r_2 = \frac{1.5 - 18.375\varepsilon}{1 + \varepsilon}; \quad r_3 = \frac{6.625}{1 + \varepsilon}; \quad r_4 = \frac{6.625}{1 + \varepsilon}.$$

Since, $r_3 = r_4$, the checks of nurses $D_{k,3}$ and $D_{k,4}$ must be ordered lexicographically. Ordering all of the r_i' s leads to the following conclusion.

Conclusion. The resident should check the 4 nurses in the following order: $D_{k,4}$, $D_{k,3}$, $D_{k,2}$, $D_{k,1}$.

5. Further Research

(1) The utility function is defined independent of the time that the procedure is being checked for error. Can we find the best checking

method while taking this factor into account? If we include into the utility function, the stage of the checking procedure that the j-th doctor is being checked, then this becomes a fixed point problem.

(2) We have assumed a given network structure. Assuming this is not given, can we find the construction which minimizes the error? For example, how many nurses should a specific resident check, and which ones?

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